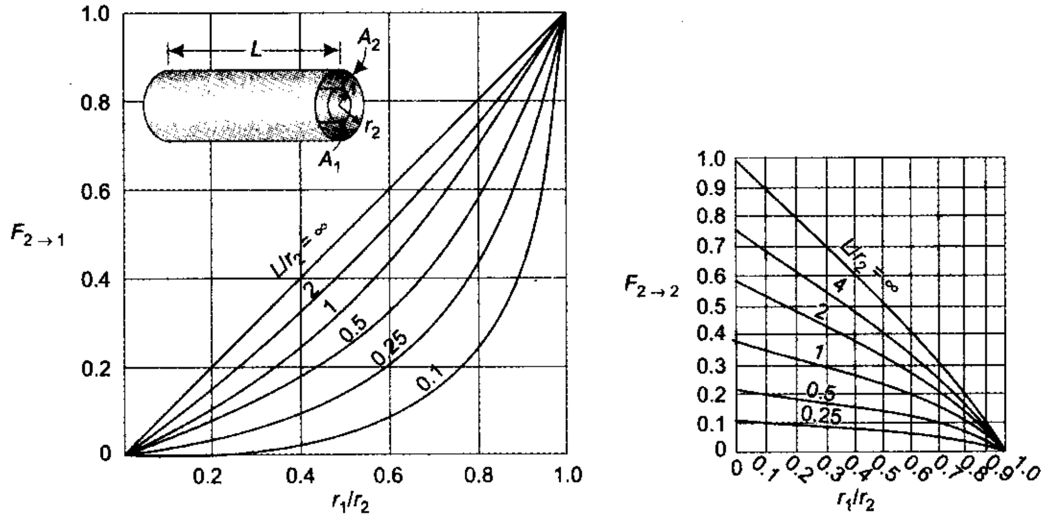

FIGURE 13.22 View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder (b) outer cylinder to itself (Source: Cengel, Yunus A. [1998]. *Heat Transfer: A Practical Approach*. Pub.: McGraw-Hill)

$$R_i := \frac{r_i}{L} \quad R_j = \frac{r_j}{L} \quad S(R_i, R_j) = 1 + \frac{1+R_j^2}{R_i^2}$$

$$F_{12'}(R_i \cdot R_j) := \frac{1}{2}\cdot\left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4\cdot\left(\frac{R_j}{R_i}\right)^2\right]^{\frac{1}{2}}\right]$$

(*view factor for coaxial parallel disks*)

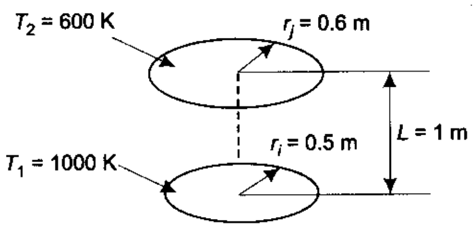

FIGURE Example 13.9 Coaxial parallel disks

Here, first, S is written as a function of R_i and R_j where, $R_i = r_i/L$ and $R_j = r_j/L$. Then, F_{12} is expressed as a function of R_i and R_j. Now, F_{12} is easily obtained for ***any*** values of R_i and R_j by simply writing $F_{12}\cdot(R_i, R_j) =$.

Therefore, in this case, $R_i = 0.5$

and, $R_j = 0.6$

We get: $F_{12}(0.5, 0.6) = 0.232$

Verify This result may be verified from Fig. 13.20 where, F_{12} is plotted against L/r_i for various values of r_j/L. Now, for our problem, $L/r_i = 1/0.5 = 2$, and $r_j/L = 0.6/1 = 0.6$. Then, from Fig. 13.20, we read $F_{12} = 0.232$, approximately,

i.e. $F_{12} := 0.232$

Therefore, net transfer between disks 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) \text{ W} \qquad (\textit{from Eq. 13.40})$$

i.e. $Q_{net} = 8.992 \times 10^3$ **W.**

13.6.3 By Use of View Factor Algebra

Often, we have to find out view factors for geometries for which readily no analytical relations or graphs are available. In such cases, sometimes, it may be possible to get the required view factor in terms of view factors of already known geometries, by suitable manipulation using view factor algebra. For this purpose, we remember the definition of view factor (as the fraction of energy emitted by surface 1 and directly falling on surface 2), and invoke the summation rule, reciprocity relation, and inspection of geometry.

We shall illustrate this procedure with some important examples:

Example 13.10. Find out the net heat transferred between the areas A_1 and A_2 shown in Fig. Example 13.10. Area 1 is maintained at 700 K, and area 2 is maintained at 400 K . Assume both the surfaces to be black.

Solution. This is the case of heat transfer between two black surfaces. So, we use Eq. 13.40 i.e.

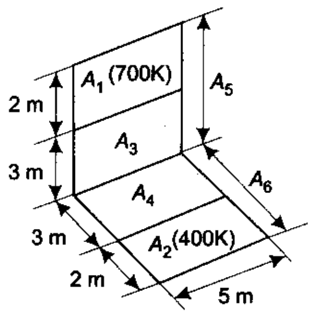

FIGURE Example 13.10 Perpendicular rectangles with a common edge

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) = A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4), \text{ W} \quad ...(13.40)$$

So, the problem reduces to calculating the view factor F_{12} or F_{21}. We see that to calculate F_{12} for areas A_1 and A_2 as oriented in the Fig. Example 13.10 we do not readily have an analytical relation or a graph. Let us denote the combined areas $(A_1 + A_3)$ by A_5 and $(A_2 + A_4)$ by A_6. Then, we see that A_5 and A_6 are perpendicular rectangles which have a common edge, and we have graphs or analytical relation for the view factor for such an orientation. Then, we resort to view factor algebra, as follows:

Remember that by definition, view factor F_{12} is the fraction of radiant energy emitted by surface 1 which falls directly on surface 2. Looking at the Fig. Example 13.10 we can say that fraction of energy leaving A_1 and falling on A_2 is equal to the fraction falling on A_6 minus the fraction falling on A_4.

i.e. $F_{12} = F_{16} - F_{14}$ *(by definition of view factor)*

i.e. $F_{12} = F_{61} \cdot \frac{A_6}{A_1} - F_{41} \cdot \frac{A_4}{A_1}$ *(since by reciprocity relation, $A_1 \cdot F_{16} = A_6 \cdot F_{16}$, and $A_1 \cdot F_{14} = A_4 \cdot F_{41}$.)*

i.e. $F_{12} = \frac{A_6}{A_1} \cdot (F_{65} - F_{63}) - \frac{A_4}{A_1} \cdot (F_{45} - F_{43})$ *(Eq. A ... using the definition of view factor, as done in first step above)*

Now, observe that view factors F_{65}, F_{63}, F_{45} and F_{43} refer to perpendicular rectangles with a common edge, and can be readily obtained from Fig. 13.21, or by analytical relation given in Table 13.5.

We re-write the view factor relation for perpendicular rectangles with a common edge, given in Table 13.5 as follows, for ease of calculation with Mathcad:

$$H := \frac{Z}{X} \quad W := \frac{Y}{X} \quad A(W) := \frac{1}{\pi \cdot W} \quad B(W) = W \cdot \text{atan}\left(\frac{1}{W}\right)$$

$$C(H) := H \cdot \text{atan}\left(\frac{1}{H}\right) \quad D(H, W) := (H^2 + W^2)^{\frac{1}{2}} \cdot \text{atan}\left[\frac{1}{(H^2 + W^2)^{\frac{1}{2}}}\right]$$

$$E(H, W) := \frac{(1+W^2)\cdot(1+H^2)}{(1+W^2+H^2)} \cdot \left[\frac{W^2\cdot(1+W^2+H^2)}{(1+W^2)\cdot(W^2+H^2)}\right]^{W^2} \cdot \left[\frac{H^2\cdot(1+H^2+W^2)}{(1+H^2)\cdot(H^2+W^2)}\right]^{H^2}$$

$$F_{ij}(H, W) := A(W) \cdot \left(B(W) + C(H) - D(H, W) + \frac{1}{4} \cdot \ln(E(H, W))\right)$$

(Eq. B...*view factor for coaxial perpendicular rectangles with a common edge)*

To find F_{65}:

$X := 5 \quad Y := 5 \quad Z := 5$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.12)*

$H := \frac{Z}{X}$ i.e. $H = 1$

$W := \frac{Y}{X}$ i.e. $W = 1$

Therefore,

$F_{ij}(1, 1) = 0.2$ *(substituting in Eq. B)*

i.e. $F_{65} := 0.2$ *(view factor from area A_6 to A_5)*

Note: This value can be verified from Fig. 13.21 also.

To find F_{63}:

$X := 5 \quad Y := 5 \quad Z = 3$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.12)*

$H := \frac{Z}{X}$ i.e. $H = 0.6$

$W := \frac{Y}{X}$ i.e. $W = 1$

Therefore,

$F_{ij}(0.6, 1) = 0.161$ *(substituting in Eq. B)*

i.e. $F_{63} := 0.161$ *(view factor from area A_6 to A_3)*

Note: This value also can be verified from Fig. 13.21.

To find F_{45}:

$X := 5 \quad Y := 3 \quad Z := 5$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.10)*

$H := \frac{Z}{X}$ i.e. $H = 1$

$W := \frac{Y}{X}$ i.e. $W = 0.6$

Therefore,

$F_{ij}(1, 0.6) = 0.269$ *(substituting in Eq. B)*

i.e. $F_{45} := 0.269$ *(view factor from area A_4 to A_5)*

Note: This value also can be verified from Fig. 13.21.

To find F_{43}:

$X := 5 \quad Y := 3 \quad Z := 3$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.10)*

$H := \frac{Z}{X}$ i.e. $H = 0.6$

$W := \frac{Y}{X}$ i.e. $W = 0.6$

Therefore,

$F_{ij}(0.6, 0.6) = 0.231$ *(substituting in Eq. B)*

i.e. $F_{43} := 0.231$ *(view factor from area A_4 to A_3)*

Note: This value also can be verified from Fig. 13.21.

Areas:

From Fig. Example 13.10, we have:

$A_1 := 10 \quad A_2 := 10 \quad A_3 := 15 \quad A_4 := 15$

$A_5 := 25 \quad A_6 := 25 \text{ m}^2$

Then, from Eq. A:

$$F_{12} := \left[\frac{A_6}{A_1}\cdot(F_{65} - F_{63}) - \frac{A_4}{A_1}\cdot(F_{45} - F_{43})\right]$$

i.e. $F_{12} := 0.041$ *(view factor from A_1 to A_2.)*

Note: F_{21} can be calculated, if required, by reciprocity relation, i.e. $A_1.F_{12} = A_2.F_{21}$

Therefore, net heat transfer between surfaces 1 and 2:

$$Q_{net} = A_1\cdot F_{12}\cdot \sigma\cdot(T_1^4 - T_2^4) \text{ W} \quad \textit{(from Eq. 13.40)}$$

Here, we have:

$T_1 := 700$ K *(temperature of surface A_1)*

$T_2 := 400$ K *(temperature of surface A_2)*

$\sigma := 5.67 \times 10^{-8}$ W/m^{2}K *(Stefan–Boltzmann constant)*

Therefore,

$$Q_{net} := A_1\cdot F_{12}\cdot \sigma\cdot(T_1^4 - T_2^4)$$

i.e. $Q_{net} = 4.926 \times 10^3$ **W.**

Example 13.11. Find out the relevant view factors for the geometries shown in Fig. Example 13.11:

(a) a long tube with cross section of an equilateral triangle
(b) a black body completely enclosed by another black body
(c) diagonal partition inside a long square duct
(d) sphere of diameter d inside a cubical box of sides, $L = d$
(e) hemispherical surface closed by a plane surface, and
(f) the end and surface of a circular cylinder whose length is equal to diameter.

Solution. General principle in solving these problems is to invoke: Summation rule, reciprocity theorem, inspection of geometry for symmetry, and of course, remembering the definition of view factor:

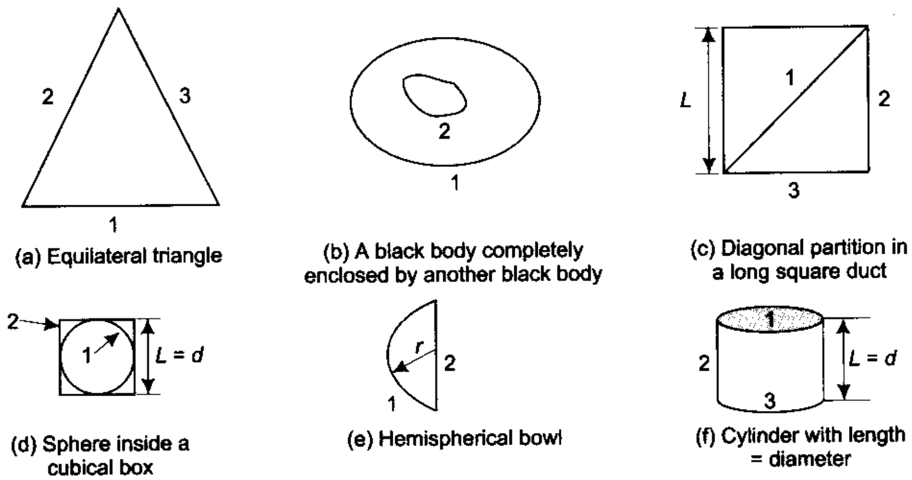

FIGURE Example 13.11 Different geometries

(a) **Long tube with cross section of equilateral triangle: See Fig. 13.11a.**

For surface 1: $F_{11} + F_{12} + F_{13} = 1$ *(summation rule)*

But, $F_{11} = 0$ *(since surface 1 is flat and cannot 'see' itself.)*

Therefore, $F_{12} + F_{13} = 1$

Now, by inspection of geometry, we find that surfaces 2 and 3 are located symmetrically w.r.t. surface 1, since it is an equilateral triangle. Therefore, radiation from surface 1 is divided equally between surfaces 2 and 3.

i.e. $F_{12} = F_{13} = 0.5$

Similarly, for surface 2, we write:

$$F_{21} + F_{23} = 1$$

i.e. $F_{23} = 1 - F_{21}$

But, $F_{21} = \frac{A_1}{A_2} \cdot F_{12} = F_{12}$ *(since $A_1 = A_2$)*

Therefore, $F_{23} = 1 - F_{12} = 0.5$

Similarly, for surface 3.

(b) **Black body enclosed inside a black enclosure: See Fig. Ex. 13.11b**

For surface 1: $F_{11} + F_{12} = 1$ *(by summation rule)*

and, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ *(by reciprocity)*

i.e. $F_{12} = \frac{A_2}{A_1} \cdot F_{21}$

Now, $F_{11} = 1 - F_{12}$

i.e. $F_{11} = 1 - \frac{A_2}{A_1} \cdot F_{21}$

But, $F_{21} = 1$ *(since all the energy radiated by surface 2 is directly intercepted by surface 1.)*

Therefore, $F_{11} = 1 - \frac{A_2}{A_1}$.

(c) **Diagonal partition within a long square duct: See Fig. Ex. 13.11c.**

For surface 1: $F_{11} + F_{12} + F_{13} = 1$ *(by summation rule)*

But, $F_{11} = 0$ *(since surface 1 is flat and cannot 'see' itself.)*

Therefore, $F_{12} + F_{13} = 1$

By symmetry: $F_{13} = F_{12} = 0.5$ *(since radiation emitted by surface 1 is divided equally between surfaces 2 and 3)*

By reciprocity: $F_{21} = \frac{A_1}{A_2} \cdot F_{12}$

i.e. $$F_{21} = \frac{\sqrt{2}\cdot L}{L}\cdot 0.5 = 0.71.$$

(d) Sphere inside a cubical box: See Fig. Ex. 13.11d

For surface 1: $F_{11} = 0$ *(since surface of the sphere is convex and cannot 'see' itself.)*

And, $F_{11} + F_{12} = 1$ *(by summation rule)*

Therefore, $F_{12} = 1$

By reciprocity $$F_{21} = \frac{A_1}{A_2}\cdot F_{12}$$

i.e. $$F_{21} = \frac{\pi\cdot d^2}{6\cdot d^2} = \frac{\pi}{6}$$ *...since L = d*

i.e. $F_{21} = 0.524.$

(e) Hemispherical surface closed by a flat surface: See Fig. Ex. 13.11e.

For surface 1: $F_{11} + F_{12} = 1$ *(summation rule)*

Also, $F_{21} = 1$ *(since surface 2 is flat and cannot 'see' itself, and all radiation emitted by surface 2 falls directly on the hemisperical surface 1.)*

By reciprocity: $$F_{12} = \frac{A_2}{A_1}\cdot F_{21}$$

i.e. $$F_{12} = \frac{\pi\cdot r^2}{2\cdot\pi\cdot r^2}\cdot 1 = 0.5$$

Therefore, $F_{11} = 1 - F_{12} = 0.5.$

(f) End and sides of a circular cylinder (L = d): See Fig. Ex. 13.11f.

From the Fig. note that the two end surfaces are denoted by 1 and 3 and the side surface is denoted by 2.

View factor F_{13}: Surfaces 1 and 3 can be considered as two concentric parallel disks. Therefore, F_{13} can be found out from Fig. 13.20 or by analytical relation given in Table 13.5. Let us use the analytical relation:

We have:

$$R_i := \frac{r_i}{L} \quad R_j = \frac{r_j}{L} \quad S(R_i, R_j) = 1 + \frac{1+R_j^2}{R_i^2}$$

$$F_{ij}(R_i, Rj) := \frac{1}{2}\cdot\left[S(R_i,R_j) - \left[S(R_i,R_j)^2 - 4\cdot\left(\frac{R_j}{R_i}\right)^2\right]^{\frac{1}{2}}\right]$$ *(view factor for coaxial parallel disks)*

Now, for a cylinder with $L = d$:

$R_i := 0.5 \quad R_j = 0.5$

Therefore, $F_{ij}(R_i, R_j) = 0.172$

i.e. $F_{13} = 0.172$ *(view factor from surface 1 to surface 3)*

For surface 1: $F_{11} + F_{12} + F_{13} = 1$ *(by summation rule)*

But, $F_{11} = 0$ *(since surface 1 is flat and cannot 'see' itself.)*

Therefore, $F_{12} := 1 - F_{13}$

i.e. $F_{12} = 0.828$

By reciprocity: $$F_{21} = \frac{A_1}{A_2}\cdot F_{12}$$

i.e. $$F_{21} = \frac{\frac{\pi\cdot d^2}{4}}{\pi\cdot d\cdot L}\cdot 0.828$$

i.e. $$F_{21} = \frac{\frac{\pi\cdot d^2}{4}}{\pi\cdot d^2}\cdot 0.828$$ *(since L = d)* i.e. $$F_{21} = \frac{0.828}{4} = 0.207$$

Also, by symmetry: $F_{32} = F_{12} = 0.828$

and, $F_{23} = F_{21} = 0.207.$

Example 13.12. Find out the view factor (F_{11}) of a cavity with respect to itself. Hence, find out the view factor F_{11} for the following:

(a) a cylindrical cavity of diameter '*d*' and depth '*h*'

(b) a conical cavity of diameter '*d*' and depth '*h*'

(c) a hemispherical bowl of diameter '*d*'.

Solution. See Fig. Example 13.12

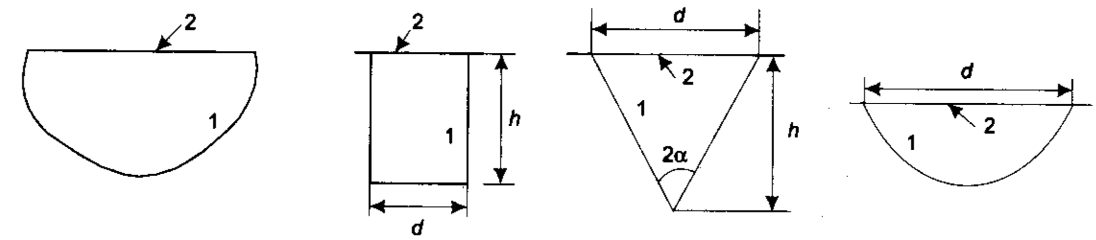

FIGURE Example 13.12 View factors for cavities

View factor of a general cavity w.r.t. itself:

See Fig. Example 13.12a.

We desire to find F_{11}. It is obvious from the Fig. Example 13.12 that part of the radiation emitted by the cavity surface 1, falls on itself and therefore, F_{11} exists.

Close the opening (or mouth) of the cavity by a hypothetical flat surface 2. Then, surfaces 1 and 2 together form an enclosure. We can write:

For surface 1: $F_{11} + F_{12} = 1$ *((a)...by summation rule)*

For surface 2: $F_{21} + F_{22} = 1$ *(by summation rule)*

But, $F_{22} = 0$ *(since surface 2 is flat and cannot 'see' itself.)*

Therefore, $F_{21} = 1$

Further, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ *(by reciprocity)*

i.e. $$F_{12} = \frac{A_2}{A_1}$$

Now, $F_{11} = 1 - F_{12}$ *(from Eq. a)*

i.e. $$F_{11} = 1 - \frac{A_2}{A_1} \qquad (Eq.\ b)$$

Eq. b is an important result, since it gives the shape factor of any general cavity w.r.t. itself.

Now, this result will be applied to following specific cavities:

(a) **F_{11} for a cylindrical cavity of diameter '*d*' and depth '*h*': See Fig. Example 13.12b.**

We have: $F_{11} = 1 - \frac{A_2}{A_1}$

i.e. $$F_{11} = 1 - \frac{\frac{\pi \cdot d^2}{4}}{\frac{\pi \cdot d^2}{4} + \pi \cdot d \cdot h}$$ *(Note that A_1 consists of the area of bottom circular surface and the cylindrical side surfaces)*

$$= 1 - \frac{d}{d + 4 \cdot h}$$

i.e. $F_{11} = \dfrac{4 \cdot h}{4 \cdot h + d}$.

(b) F_{11} for a conical cavity of diameter 'd' and depth 'h': See Fig. Example 13.12c.

We have: $F_{11} = 1 - \dfrac{A_2}{A_1}$

i.e. $F_{11} = 1 - \dfrac{\dfrac{\pi \cdot d^2}{4}}{\dfrac{\pi \cdot d \cdot L}{2}}$ *(where, L is the slant height of the cone)*

i.e. $F_{11} = 1 - \dfrac{d}{2 \cdot L}$

i.e. $F_{11} = 1 - \sin(\alpha)$ *(where, α is the half-vertex angle of the cone.)*

Alternatively:

To get F_{11} in terms of depth 'h', we write:

We have: $F_{11} = 1 - \dfrac{d}{2 \cdot L}$

i.e. $F_{11} = 1 - \dfrac{d}{2 \cdot \sqrt{h^2 + \dfrac{d^2}{4}}}$

i.e. $F_{11} = 1 - \dfrac{d}{\sqrt{4 \cdot h^2 + d^2}}$

(c) F_{11} for a hemispherical bowl of diameter 'd': See Fig. Example 13.12d.

We have: $F_{11} = 1 - \dfrac{A_2}{A_1}$

i.e. $F_{11} = 1 - \dfrac{\dfrac{\pi \cdot d^2}{4}}{\dfrac{\pi \cdot d^2}{2}}$

i.e. $F_{11} = 1 - \dfrac{1}{2}$

i.e. $F_{11} = 0.5$.

This result means that for any hemispherical cavity, half of the radiation emitted by the surface 1 falls on itself; it also means that the remaining half falls on the closing surface 2.

13.6.4 By Graphical Techniques

In some cases, it is possible to get view factors for some geometries by some simple graphical construction. Let us illustrate the principle of this method as follows: (See Fig. 13.23).

dA_1 and dA_2 are two differential areas at a distance r as shown. ϕ_1 and ϕ_2 are the angles made by the normals to dA_1 and dA_2 with the line connecting dA_1 and dA_2.

Now, let us make a graphical construction as follows: Construct a hemisphere of radius equal to unity with dA_1 as the centre, and project the element dA_2 on the surface of this hemisphere; this projection is shown as dA_3 in the Fig. 13.23 From geometry, we know that $dA_3 = dA_2 \cdot \cos(\phi_2)/r^2$. Next, project dA_3 on the tangential plane drawn through dA_1, i.e. on the base of the hemisphere. This projection is dA_4 in the figure above. Again, dA_4 is equal to dA_3 multiplied by the cosine of the angle ϕ_1 formed between the two projections. Thus, we have:

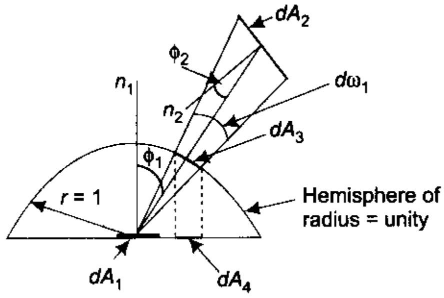

FIGURE 13.23 Graphical determination of view factor between two differential areas dA_1 and dA_2

$$dA_4 = dA_2 \cdot \frac{\cos(\phi_1)\cdot\cos(\phi_2)}{r^2}$$

Now, base of the hemisphere is a circle of unity radius, whose area is equal to π. Therefore, area dA_4 divided by the area of circle of unity radius is:

$$\frac{dA_4}{\pi} = dA_2 \cdot \frac{\cos(\phi_1)\cdot\cos(\phi_2)}{\pi \cdot r^2}$$

Now, recollect that we have already proved the view factor between two differential areas dA_1 and dA_2 to be:

$$F_{dA_1\_dA_2} = \frac{\cos(\phi_1)\cdot\cos(\phi_2)\cdot dA_2}{\pi \cdot r^2} \qquad ...(13.31)$$

i.e. from the above two expressions, it is clear that view factor from dA_1 to dA_2 is given as the ratio of two areas viz. dA_4 to π, where dA_4 is the projected area of dA_3 on the base of the hemisphere and dA_3 is the projection of dA_2 on the surface of the hemisphere of unity radius.

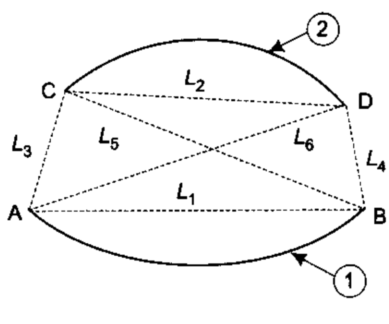

FIGURE 13.24 Crossed strings method to determine view factor between two infinitely long surfaces

If the view factor is desired from a differential area dA_1 to a finite area A_2 (instead of to a differential area dA_2), the same procedure is followed: area A_2 is projected over the surface of the hemisphere of unity radius, and the resulting area is further projected on the base of the hemisphere, and then the view factor is computed as the ratio of this projected area to π.

Many graphical and optical integrators have been developed to find out view factors between two surfaces, based on this principle.

However, above method is difficult to apply for the case of complex geometries. In such cases, experimental techniques have been adopted with success, using scale models, the underlying principle being: 'view factors of geometrically similar systems are identical'.

Hottel's crossed strings method This is an extremely simple method to find out the view factors between infinitely long surfaces; generally, channels and ducts which have a constant cross section and are very long can be modelled as two-dimensional and infinitely long. Consider Fig. 13.24:

A, B and C, D are the end points of two surfaces 1 and 2. These are connected by tightly stretched strings as shown. Then, the view factor F_{12} between surfaces 1 and 2 is given by:

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2 \cdot L_1}$$

i.e.

$$F_{12} = \frac{\Sigma(\text{Crossed strings}) - \Sigma(\text{Uncrossed strings})}{2 \cdot (\text{string on surface 1})} \qquad ...(13.46)$$

Note that this method can be applied even when the two surfaces 1 and 2 have a common edge (as in the case of a triangle); then, the common edge is treated as an imaginary string of zero length. Also, note that surfaces 1 and 2 may be curved surfaces, but L_1 and L_2 are the straight lengths connecting the edges of the respective surfaces.

Table 13.4 gives view factors for a few two-dimensional geometries.

Example 13.13. Find out the view factors F_{12} and F_{21} for two infinitely long, parallel planes whose centre lines are on the same vertical line, as shown in Fig. Example 13.13. Plate 1 is 1 m wide, plate 2 is 0.5 m wide and they are spaced 0.6 m apart.

Solution.

Data:

$L_1 := 1$ m $L_2 := 0.5$ m $S := 0.6$ m

To apply crossed strings method, we calculate L_3, L_4, L_5 and L_6:

$$L_3 := \sqrt{S^2 + \left(\frac{L_1 - L_2}{2}\right)^2}$$

i.e. $L_3 = 0.65$ m

and, $L_4 := L_3$ (*by symmetry*)

$$L_5 := \sqrt{S^2 + 0.75^2}$$

i.e. $L_5 = 0.96$ m

and, $L_6 := L_5$ (*by symmetry*)

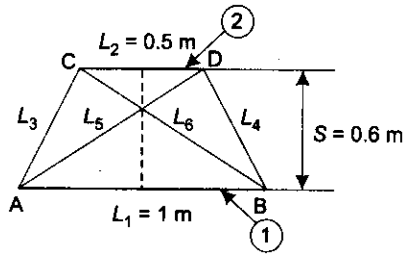

FIGURE Example 13.13 Crossed strings method to determine view factor between two infinitely long surfaces

Now, we have:

$$F_{12} = \frac{\Sigma(\text{Crossed strings}) - \Sigma(\text{Uncrossed strings})}{2\cdot(\text{string on surface 1})} \quad ...(13.46)$$

i.e.
$$F_{12} := \frac{(L_5 + L_6) - (L_3 + L_4)}{2\cdot L_1}$$

i.e. $F_{12} = 0.31$ (*view factor from surface 1 to surface 2*)

Similarly,
$$F_{21} := \frac{(L_5 + L_6) - (L_3 + L_4)}{2\cdot L_2}$$

i.e. $F_{21} = 0.621$ (*view factor from surface 2 to surface 1.*)

Alternatively:

We an use the ready formula given in Table 13.4,

$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{\frac{1}{2}} - [(W_j - W_i)^2 + 4]^{\frac{1}{2}}}{2\cdot W_i}$$ (*parallel plates with midlines connected by perpendicular.*)

In the above formula, notations are with reference to Fig. 13.17a. In the present case, according to the notation of Fig. Example 13.13, *i* stands for plate 2 and *j* stands for plate 1, and the spacing *L* stands for *S*.

i.e.
$w_i := L_2$
$w_j := L_1$
$L := S$

$W_i := \frac{w_i}{L}$ i.e. $W_i = 0.833$

$W_j := \frac{w_i}{L}$ i.e. $W_j = 1.667$

Then,
$$F_{21} := \frac{[(W_i + W_j)^2 + 4]^{\frac{1}{2}} - [(W_j - W_i)^2 + 4]^{\frac{1}{2}}}{2\cdot W_i}$$

i.e. $F_{21} = 0.621$ (*view factor from surface 2 to surface 1...same as obtained earlier.*)

13.7 Radiation Heat Exchange between Grey Surfaces

So far, we considered radiation heat exchange between black bodies. This was relatively simple since a black body absorbs all the energy falling on it and none is reflected. However, in the case of grey bodies, absorptivity is less than unity and the effect of multiple reflections has to be taken into account, and this makes the analysis more complex.

Generally, there are three methods to deal with the problem of radiation heat exchange between grey bodies:

(i) The reflection method
(ii) The electrical network method, and
(iii) The absorption factor method.

Of these, the reflection method is applied to the simplest of cases, where the number of reflections between the interacting surfaces is finite, or when the surfaces are black. The electrical network method is applied to cases of moderate complexity where the number of reflections involved are infinite, but the number of surfaces involved are not more than four or five; this method is very simple, since the standard techniques of solving electrical networks are applied to solve the equivalent thermal networks. The absorption factor method is used to solve radiation problems that can be graded as 'difficult'; here, the resulting system of linear algebraic equations have to be solved by the standard mathematical techniques (such as: matrix methods or using standard computer library programs).

Whatever the method followed, following assumptions are made to simplify the solution:

(i) All the surfaces of the enclosure are opaque ($\tau = 0$), diffuse and grey
(ii) Radiative properties such as ρ, ε and α are uniform and independent of direction and frequency
(iii) Irradiation and heat flux leaving each surface are uniform over the surface
(iv) Each surface of the enclosure is isothermal, and
(v) The enclosure is filled with a non-participating medium (such as vacuum or air).

In this book, we shall discuss only the 'electrical network method', since it is simple to apply and gives a physical 'feel' of the problem. However, before we proceed with the discussion of electrical network method, we shall study a special case of radiative heat transfer between small grey bodies.

13.7.1 Radiation Exchange between Small, Grey Surfaces

Let us consider radiative heat exchange between two small, grey bodies, 1 and 2. By 'small', we mean that their size is very small compared to the distance between them. Let the emissivities of surfaces 1 and 2 be ε_1 and ε_2, respectively, and their absorptivities be α_1 and α_2, respectively. Since the surfaces are grey (not black), surely we have to consider the effect of multiple reflections; however, implication of 'small' body is that the portion of radiation emitted by either body that is reflected by the other body is considered to be 'lost' in space and does not return to the originating surface.

Then, we write:

Energy emitted by body 1 and incident on body 2 $= F_{12}.A_1.\varepsilon_1.\sigma.T_1^4$

Of this energy, amount absorbed by body 2 $= \alpha_2.F_{12}.A_1.\varepsilon_1.\sigma.T_1^4$

Therefore, energy transferred from body 1 to body 2:

$$Q_1 = \varepsilon_1 \cdot \varepsilon_2 \cdot A_1 \cdot F_{12} \cdot \sigma \cdot T_1^4 \qquad \textit{(since } \alpha_2 = \varepsilon_2 \textit{ by Kirchhoffs law)}$$

Similarly, energy transferred from body 2 to body 1 is:

$$Q_2 = \varepsilon_1 \cdot \varepsilon_2 \cdot A_2 \cdot F_{21} \cdot \sigma \cdot T_2^4$$

and, net radiant energy exchange between 1 and 2 is:

$$Q_{12} = \varepsilon_1 \cdot \varepsilon_2 \cdot A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) = \varepsilon_1 \cdot \varepsilon_2 \cdot A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4) \qquad ...(13.47)$$

since $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ *(by reciprocity)*

The product, $(\varepsilon_1.\varepsilon_2)$ is known as 'equivalent emissivity (ε_{eq})' for a system of two 'small' grey bodies.

13.7.2 The Electrical Network Method

This method, introduced by Oppenheim in the 1950s, is simple and direct; it emphasises on the physics of the problem, and is easy to apply. Before we introduce this method, let us define two new quantities, namely irradiation and radiosity: (See Fig. 13.25).

Irradiation, (*G*) is the total radiation incident upon a surface per unit time, per unit area (W/m^2).

Radiosity, (*J*) is the total radiation leaving a surface, with no regard for its origin (i.e. reflected *plus* emitted from the surface) per unit time, per unit area (W/m^2).

Now, from Fig. 13.25, it is clear that total radiation leaving the surface (i.e. radiosity, J) is:

$$J = \rho \cdot G + \varepsilon \cdot E_b$$

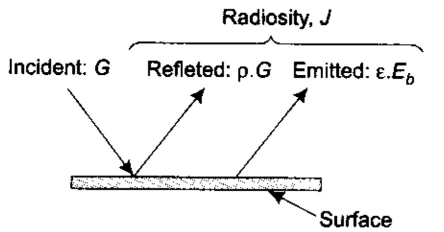

FIGURE 13.25 Irradiation and radiosity

For a grey, opaque ($\tau = 0$) surface, we have:

$$\rho = (1 - \alpha) = (1 - \varepsilon) \qquad \textit{(from Kirchhoff's law)}$$

Therefore,

$$J = (1 - \varepsilon) \cdot G + \varepsilon \cdot E_b$$

or,

$$G = \frac{(J - \varepsilon \cdot E_b)}{(1 - \varepsilon)}$$

Now, net rate of radiation energy transfer from the surface is given by: (rate of radiation energy leaving the surface minus the rate of radiation energy incident on the surface), i.e.

$$\frac{Q}{A} = J - G$$

i.e.
$$\frac{Q}{A} = J - \left(\frac{J - \varepsilon \cdot E_b}{1-\varepsilon}\right)$$

Therefore,

$$Q = \left(\frac{\varepsilon \cdot A}{1-\varepsilon}\right)\cdot(E_b - J)$$

i.e.
$$Q = \frac{E_b - J}{\frac{(1-\varepsilon)}{A\cdot\varepsilon}}, \text{ W.} \qquad \text{...(13.48)}$$

By analogy with Ohm's law, we can think of Q in Eq. 13.48 as a current flowing through a potential difference $(E_b - J)$, and the factor $(1 - \varepsilon)/A.\varepsilon$ as the resistance. Now, this resistance is the resistance to the flow of radiant heat due to the nature of the surface and is known as **'surface resistance (*R*)'**.
i.e.

$$R = \frac{(1-\varepsilon)}{A\cdot\varepsilon} \qquad \textit{(surface resistance)}$$

Surface resistance for a surface i is shown schematically in Fig. 13.26a.

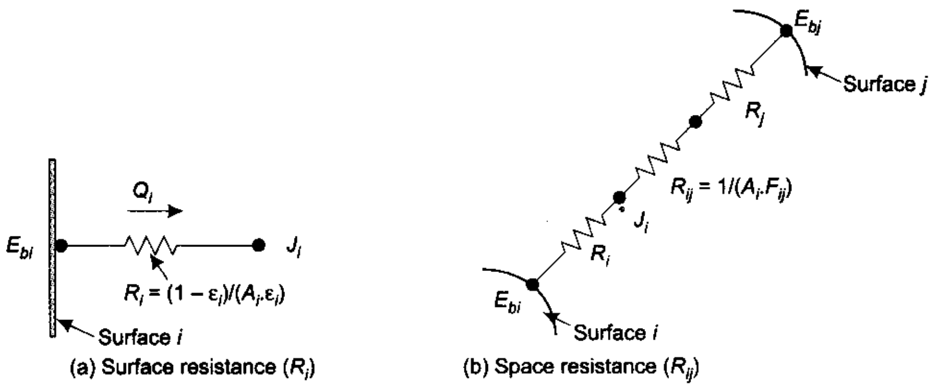

FIGURE 13.26 Surface resistance and space resistance

From Eq. 13.48, we see that if $E_{bi} > J_i$, i.e. if the emissive power is greater than the radiosity, then Q_i will be positive, which means that the net heat transfer is *from* the surface i. On the other hand, if $E_{bi} < J_i$, i.e. if the emissive power is less than the radiosity, then Q_i will be negative, and this means that the net heat transfer is *to* the surface i.

For a black body emissivity $\varepsilon = 1$; so, the surface resistance is zero, and

$$J_i = E_{bi} \qquad \textit{(for a black body...(13.49))}$$

Also, many surfaces in numerous applications are adiabatic, i.e. well insulated, and net heat transfer through such a surface is zero, since in steady state, all the heat incident on such a surface is re-radiated. These are known as **re-radiating surfaces**. Walls of a furnace is the familiar example of a re-radiating surface. Obviously, for a re-radiating surface, $Q_i = 0$, and from Eq. 13.48 we get:

$$J_i = E_{bi} = \sigma \cdot T_i^4 \qquad \textit{(for a re-radiating surface...(13.50))}$$

Note that the temperature of a re-radiating surface can be calculated from the above equation; further, note that this temperature is independent of the emissivity of the surface.

Again, consider two diffuse, grey and opaque surfaces i and j, maintained at uniform temperatures T_i and T_j, exchanging heat with each other. Then, remembering the definitions of radiosity and view factor, we can write for the radiation leaving surface i that strikes surface j:

$$Q_i = A_i \cdot F_{ij} \cdot J_i$$

Similarly, for surface j, we have:

$$Q_j = A_j \cdot F_{ji} \cdot J_j$$

Therefore, net heat interchange between surfaces i and j is:

$$Q_{ij} = A_i \cdot F_{ij} \cdot J_i - A_j \cdot F_{ji} \cdot J_j$$

i.e.
$$Q_{ij} = A_i \cdot F_{ij} \cdot (J_i - J_j) \text{ W} \quad ...(13.51)$$

since
$$A_i \cdot F_{ij} = A_j \cdot F_{ji} \quad \textit{(by reciprocity)}$$

i.e.
$$Q_{ij} = \frac{J_i - J_j}{\dfrac{1}{(A_i \cdot F_{ij})}} \text{ W}. \quad ...(13.52)$$

Again, by analogy with Ohm's law, we can write Eq. 13.52 as:

$$Q_{ij} = \frac{J_i - J_j}{R_{ij}} \text{ W}$$

where,
$$R_{ij} = \frac{1}{A_i \cdot F_{ij}} \quad ...(13.53)$$

R_{ij} is known as **'space resistance'** and it represents the resistance to radiative heat flow between the radiosity potentials of the two surfaces, due to their relative orientation and spacing.

Space resistance is illustrated in Fig. 13.26b. Note from Eq. 13.52 that if $J_i > J_j$, net heat transfer is from surface i to surface j; otherwise, the net heat transfer is from surface j to surface i.

Thus, for each diffuse, grey, opaque surface, in radiant heat exchange with other surfaces of an enclosure, there are two resistances, i.e. the surface resistance, $R_i = (1 - \varepsilon_i)/(A_i.\varepsilon_i)$, and a space resistance, $R_{ij} = 1/(A_i.F_{ij})$.

For a N surface enclosure, net heat transfer from surface i should be equal to the sum of net heat transfers from that surface to the remaining surfaces, i.e.

$$Q_i = \sum_{j=1}^{N} Q_{ij} = \sum_{j=1}^{N} A_i \cdot F_{ij} \cdot (J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \text{ W} \quad ...(13.54)$$

i.e.
$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \text{ W} \quad ...(13.55)$$

where, R_i is the surface resistance and R_{ij} is the space resistance.

This situation is shown in Fig. 13.27.

As can be seen from Fig. 13.27 rate of radiation 'current' flow to surface i through its surface resistance must be equal to the sum of all the radiation current flows from surface i to all other surfaces through the respective space resistances.

In general, there are two types of radiation problems: first (and most common), when the surface temperature T_i, and therefore, the emissive power E_{bi} is known; and, the second type is when the net radiation heat transfer at the surface i is known. Eq. 13.55 is useful in solving the first type of problems, i.e. when the surface temperature is known; instead, if the net heat transfer rate at the surface is the known quantity, Eq. 13.52 is the applicable equation. Essentially, the problem is to solve for the radiosities $J_1, J_2, ..., J_n$. As mentioned earlier, electrical network method is convenient to use if the number of surfaces in an enclosure is limited to about five; however, if the number of surfaces is more than five, the direct approach is to apply Eq. 13.55 for each surface whose temperature is known, and Eq. 13.52 for each surface at which the net heat transfer rate is known, and solve the resulting set of N linear, algebraic equations for the N unknowns, namely, $J_1, J_2, ..., J_n$

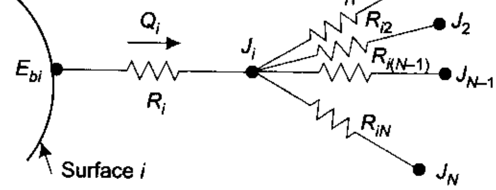

FIGURE 13.27 Radiation heat transfer from surface i to other surfaces in a N-surface enclosure

by standard mathematical methods. Once the radiosities are known, Eq. 13.48 may be applied to determine either the heat transfer rate or the temperature, as the case may be.

13.7.3 Radiation Heat Exchange in Two-zone Enclosures

Now, with the background of above discussion on the surface resistance and space resistance in connection with diffuse, grey, opaque surfaces, let us consider the radiant heat transfer in a two-zone enclosure. This simply means that the two surfaces, together, make up the enclosure and 'see' only themselves and nothing else. Many, practically important geometries may be classified as two-zone enclosures, e.g. a small body enclosed by a large body, a pipe passing through a large room, concentric spheres, concentric long cylinders, long, parallel plane surfaces, etc.

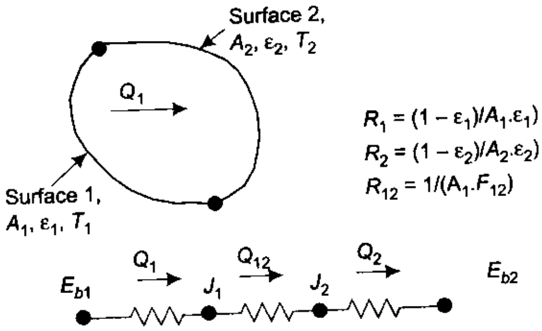

FIGURE 13.28 Two-surface enclosure and its radiation network

Fig. 13.28 shows a schematic of a typical two-zone enclosure and the associated radiation (or, thermal) network.

Surfaces 1 and 2 forming the enclosure are diffuse, grey and opaque. Let their emissivities, temperatures and areas be (ε_1, T_1, A_1) and (ε_2, T_2, A_2), respectively. The radiation network is shown in Fig. 13.28. Each surface has one surface resistance associated with it and there is one space resistance between the two radiosity potentials, and all the resistances are in series, as shown. The 'heat current' (Q_{12}) in this circuit is calculated by dividing the 'total potential' ($E_{b1} - E_{b2}$) by the 'total resistance' ($R_1 + R_{12} + R_2$). So, we write:

$$Q_{12} = Q_1 = Q_2 = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

i.e.

$$Q_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

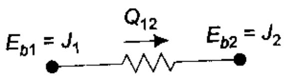

FIGURE 13.29 Radiation network for two black surfaces forming an enclosure

i.e.

$$Q_{12} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}} \text{ W.} \quad \text{...(13.56)}$$

Eq. 13.56 is an important equation, which gives net rate of heat transfer between two grey, diffuse, opaque surfaces which form an enclosure, i.e. which 'see' only each other and nothing else.

Now, let us consider a few special cases of two-surface enclosure. Basic radiation network for all these cases is the same as given in Fig. 13.28 and the basic, governing equation is Eq. 13.56, which is modified depending upon the case considered.

Case (i): Radiant heat exchange between two black surfaces:
For a black body, $\varepsilon = 1$, and $J = E_b$, as explained earlier. i.e. surface resistance [$= (1 - \varepsilon)/(A.\varepsilon)$] of a black body is zero. Then, the radiation network will consist of only a space resistance between the two radiosity potentials, as shown in Fig. 13.29:

Then, from Eq. 13.56, we get:

$$Q_{12} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{A_1 \cdot F_{12}}}$$

i.e.

$$Q_{12} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4), \text{ W} \quad \textit{(for two black surfaces forming an enclosure...(13.57))}$$

Next, we shall consider four cases of practical interest where the view factor between the inner surface 1 and the outer surface 2 (i.e. F_{12}) is equal to 1, and also the net radiation from a grey cavity.

Case (ii): Radiant heat exchange for a small object in a large cavity:
See Fig. 13.30 (a). A practical example of a small object in a large cavity is the case of a steam pipe passing through a large plant room.

For this case, we have:

$$\frac{A_1}{A_2} = 0$$

and, $$F_{12} = 1$$

And, Eq. 13.56 becomes:

$$Q_{12} = A_1 \cdot \sigma \cdot \varepsilon_1 \cdot (T_1^4 - T_2^4) \qquad \text{(for small object in a large cavity...(13.58))}$$

Case (iii): Radiant heat exchange between infinitely large parallel plates:
See Fig. 13.30 (b). In this case, $A_1 = A_2 = A$, say, and $F_{12} = 1$

Then, Eq. 13.56 becomes:

$$Q_{12} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad \text{(for infinitely large parallel plates...(13.59))}$$

Case (iv): Radiant heat exchange between infinitely long concentric cylinders:
See Fig. 13.30 (c). In this case:

$$F_{12} = 1$$

Then, Eq. 13.56 becomes:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \text{(for infinitely long concentric cylinders...(13.60))}$$

where,

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Remember that A_1 refers to the inner (or enclosed) surface.

Eq. 13.60 is known as **'Christiansen's equation'**.

Case (v): Radiant heat exchange between concentric spheres:
See Fig. 13.30 (d). In this case:

$$F_{12} = 1$$

Then, Eq. 13.56 becomes

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \text{(for concentric spheres...(13.61))}$$

where,

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Remember, again, that A_1 refers to the inner (or enclosed) surface.

Case (vi): Energy radiated from a grey cavity:
Consider a grey cavity as shown in Fig. 13.31. Let ε_1, A_1 and T_1 be its emissivity, area and temperature (in Kelvin), respectively. Now, energy will stream out of the cavity into the surrounding space through the opening (or, mouth) of the cavity. Let the opening be covered by an imaginary surface A_2. Thus, it is a two-surface enclosure. Now, since the cavity is very small compared to the space outside, practically all the energy emitted by the cavity will be absorbed by space, and it is reasonable to assume that radiation coming to the cavity from space is negligible, i.e. the space acts like a black body at a temperature of zero Kelvin as far as the cavity is

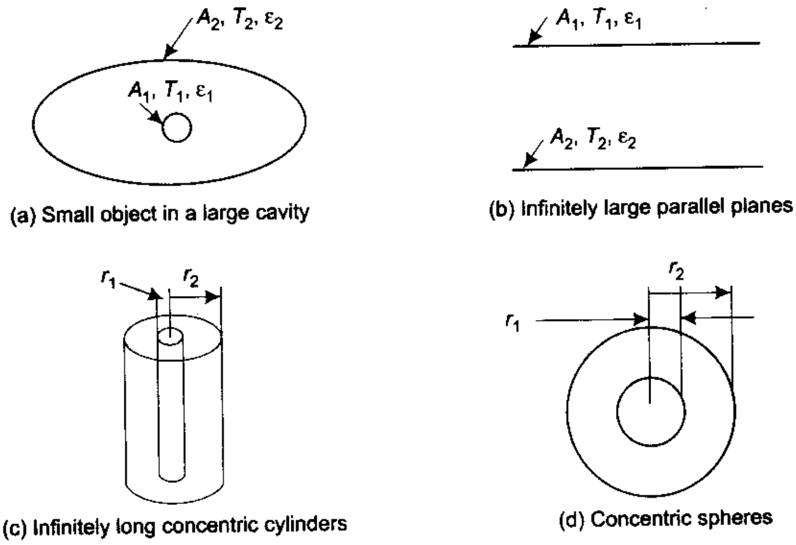

FIGURE 13.30 Few two-surface enclosures where $F_{12} = 1$

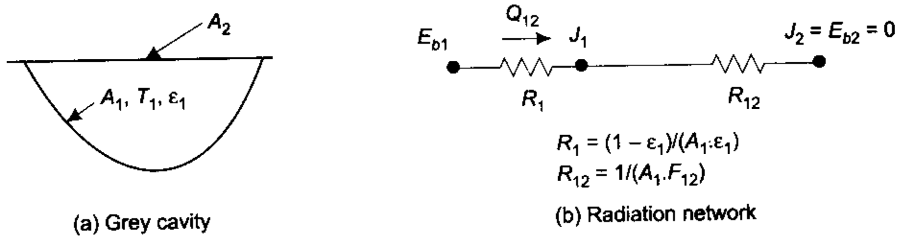

FIGURE 13.31 Radiation from a grey cavity

concerned. So, surface 2 is black at zero Kelvin for our analysis. Implication of this is that surface resistance of surface 2 is zero, and radiosity of surface 2 is equal to its emissive power, which in turn, is equal to zero since the temperature is zero Kelvin. So, the radiation network for this case will be as shown in Fig. 13.31:

Therefore, net energy radiated from the grey cavity is given by:

$$Q_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12}}$$

i.e.

$$Q_{12} = \frac{E_{b1} - 0}{R_1 + R_{12}} \qquad \textit{(since } E_{b2} = 0 \textit{ at } T_2 = \textit{zero Kelvin)}$$

i.e.

$$Q_{12} = \frac{\sigma \cdot T_1^4}{\frac{1-\varepsilon_1}{\varepsilon_1 \cdot A_1} + \frac{1}{A_1 \cdot F_{12}}}$$

Now,

$$F_{11} + F_{12} = 1 \qquad \textit{(by summation rule)}$$

i.e.

$$F_{12} = 1 - F_{11}$$

Then, Q_{12} becomes:

$$Q_{12} = \frac{\sigma \cdot T_1^4}{\frac{1-\varepsilon_1}{\varepsilon_1 \cdot A_1} + \frac{1}{A_1 \cdot (1 - F_{11})}}$$

i.e. $$Q_{12} = \frac{A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot (1 - F_{11})}{(1 - \varepsilon_1) \cdot (1 - F_{11}) + \varepsilon_1}$$

i.e. $$Q_{12} = \frac{A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot (1 - F_{11})}{1 - \varepsilon_1 - F_{11} + \varepsilon_1 \cdot F_{11} + \varepsilon_1}$$

i.e. $$Q_{12} = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot \left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}}\right] \text{ W}$$ *(net radiation from grey...(13.62))*

Eq. 13.62 is an important result, which gives net radiation from a grey cavity to surrounding space. If it is desired, for example, to calculate the net radiation from a blind hole drilled in a flange, then the relation to use is the Eq. 13.62.

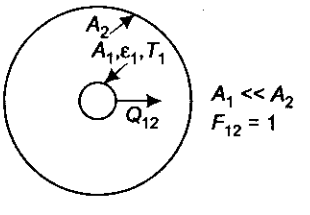

FIGURE Example 13.14 Two-surface enclosure with $A_1 << A_2$

Example 13.14. A long pipe, 50 mm in diameter, passes through a room and is exposed to air at 20 deg.C. Pipe surface temperature is 93 deg.C. Emissivity of the surface is 0.6. Calculate the net radiant heat loss per metre length of pipe.

(M.U. 1991)

Solution. The pipe is enclosed by the room; so, it is two-surface enclosure problem. Further, area of the pipe is very small, compared to the area of the room. Therefore, this is a case of a small object surrounded by a large area, and we have:

$$\frac{A_1}{A_2} = 0$$

and, $$F_{12} = 1$$

∴ $$Q_{12} = A_1 \cdot \sigma \cdot \varepsilon_1 \cdot (T_1^4 - T_2^4)$$ *(for small object in a large cavity...(13.58))*

Data:

$d_1 := 0.05$ m $L := 1$ m $\varepsilon_1 := 0.6$ $T_1 := 93 + 273$ K $T_2 := 20 + 273$ K

$\sigma := 5.67 \times 10^{-8}$ W/(m^{2}K) *(Stefan–Boltzmann constant)*

Now, $A_1 := \pi \cdot d_1 \cdot L$

i.e. $A_1 = 0.157$ m^2 *(surface area of the pipe per metre length)*

Then, applying Eq. 13.58, we get:

$Q_{12} := A_1 \cdot \sigma \cdot \varepsilon_1 \cdot (T_1^4 - T_2^4)$

i.e. $Q_{12} = 56.507$ **W** ***(net radiant heat loss from the pipe** per metre length.)*

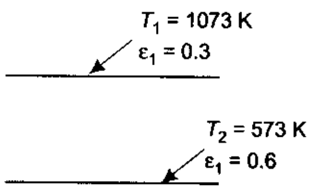

FIGURE Example 13.15 Two infinitely large parallel plates

Example 13.15. Calculate the net radiant heat interchange per m^2 for two large parallel plates maintained at 800°C and 300°C. The emissivities of two plates are 0.3 and 0.6, respectively. (M.U. 1993)

Solution. The plates are parallel to each other, and are very large; so, it is a two-surface enclosure problem, with two infinite parallel plates. We have:

Data:

$T_1 := 800 + 273$ K $T_2 := 300 + 273$ K $\varepsilon_1 := 0.3$ $\varepsilon_2 := 0.6$

$\sigma := 5.67 \times 10^{-8}$ W/(m^{2}K) *(Stefan–Boltzmann constant)*

$A := 1$ m^2 *(area of the surface)*

For infinite parallel plates, we have the relation:

$$Q_{12} := \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \text{ W}$$ *(for infinitely large parallel plates ...(13.59))*

i.e. $Q_{12} = 1.726 \times 10^4$ **W/m^2** ***(radiant heat transfer per m^2 of the plates.)***

Example 13.16. A spherical liquid oxygen tank, 0.3 m in diameter is enclosed concentrically in a spherical container of 0.4 m diameter and the space in between is evacuated. The tank surface is at −183°C and has an emissivity of 0.2. The container surface is at 25°C and has an emissivity of 0.25. Determine the net radiant heat transfer rate. (M.U.)

Solution. This is the case of two surface enclosure, with one sphere enclosed by another sphere. If 1 denotes inner sphere, and 2 outer sphere, we have for view factors: $F_{11} = 0$ and $F_{12} = 1$

Data:

$T_1 := 183 + 273$ K $\quad T_2 := 25 + 273$ K $\quad \varepsilon_1 := 0.2 \quad \varepsilon_2 := 0.25 \quad r_1 := 0.15$ m $\quad r_2 := 0.2$ m

$\sigma := 5.67 \times 10^{-8}$ W/(m²K) (*Stefan–Boltzmann constant*) $\quad A_1 := 4\cdot\pi\cdot r_1^2$ i.e. $A_1 = 0.283$ m² (*area of inner surface*)

$A_2 := 4\cdot\pi\cdot r_2^2$ i.e. $A_2 = 0.503$ m² (*area of outer surface*)

Now, for the case of concentric spheres, we have:

$$Q_{12} := \frac{A_1\cdot\sigma\cdot(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right)\cdot\left(\frac{1}{\varepsilon_2} - 1\right)}$$ (*for concentric spheres...(13.61)*)

Therefore,

$Q_{12} = -18.748$ W (***net radiant heat interchange between inner and outer spheres.***)

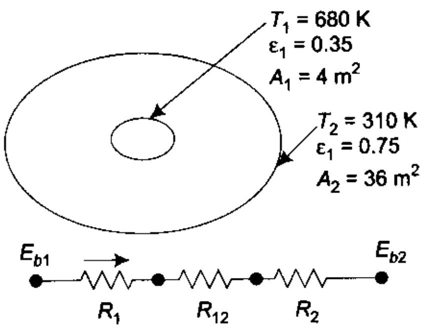

FIGURE Example 13.17 Radiation net-work for a convex grey body completely enclosed by another grey body

Note: Negative sign indicates that heat flows from outer sphere to inner sphere; this is certainly so, since the inner sphere is at a lower temperature than the outer sphere.

Example 13.17. A convex grey body having a surface area of 4 m² has $\varepsilon_1 = 0.35$ and $T_1 = 680$ K. This is completely enclosed by a grey surface having an area of 36 m², $\varepsilon_2 = 0.75$ and $T_2 = 310$ K. Find the net rate of heat transfer Q_{12} between the two surfaces. (M.U. 1999)

Solution. This is the case of a two surface enclosure. Inner surface is convex; so, view factor $F_{11} = 0$. Also, $F_{12} = 1$ since the inner body is completely enclosed by the outer surface.

The radiation network for this problem is shown in Fig. Example 13.22 below:

Data:

$T_1 := 680$ K $\quad T_2 := 310$ K $\quad \varepsilon_1 := 0.35 \quad \varepsilon_2 := 0.75$

$A_1 := 4$ m² $\quad A_2 := 36$ m²

$\sigma := 5.67 \times 10^{-8}$ W/(m²K) (*Stefan–Boltzmann constant*)

$F_{12} := 1$ (*since all the heat radiation emitted by surface* 1 *is intercepted by surface* 2.)

$E_{b1} := \sigma\cdot T_1^4$ i.e. $E_{b1} = 1.212 \times 10^4$ W/m²

$E_{b2} := \sigma\cdot T_2^4$ i.e. $E_{b2} = 523.636$ W/m²

Now, $\quad R_1 := \frac{1-\varepsilon_1}{\varepsilon_1\cdot A_1}$

i.e. $\quad R_1 = 0.464$ m⁻² (*surface resistance of inner surface*)

and, $\quad R_2 := \frac{1-\varepsilon_2}{\varepsilon_2\cdot A_2}$

i.e. $\quad R_2 = 9.259 \times 10^{-3}$ m⁻² (*surface resistance of outer surface*)

Also, $\quad R_{12} := \frac{1}{A_1\cdot F_{12}}$

i.e. $\quad R_{12} = 0.25$ m⁻² (*space resistance between inner and outer surface*)

Therefore,

$R_{tot} := R_1 + R_{12} + R_2$

i.e. $\quad R_{tot} = 0.724$ m⁻² (*total resistance between inner and outer surface*)

Then, net rate of heat transfer between surfaces 1 and 2 is given by:

$$Q_{12} := \frac{E_{b1} - E_{b2}}{R_{tot}}$$

i.e. $\quad Q_{12} = 1.603 \times 10^4$ **Watts.**

Alternatively:

We can apply the direct formula for a two surface enclosure, for which $F_{11} = 0$, $F_{12} = 1$, i.e.

$$Q_{12} := \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$ *(for convex surface enclosed by another surface...(13.61))*

i.e. $Q_{12} = 1.603 \times 10^4$ **Watt.**

Example 13.18. A hemispherical furnace of radius 1.0 m has a roof temperature of $T_1 = 800$ K and emissivity $\varepsilon_1 = 0.8$. The flat circular floor of the furnace has a temperature of $T_2 = 600$ K and emissivity $\varepsilon_2 = 0.5$. Calculate the net radiant heat exchange between the roof and the floor. (M.U. 1998)

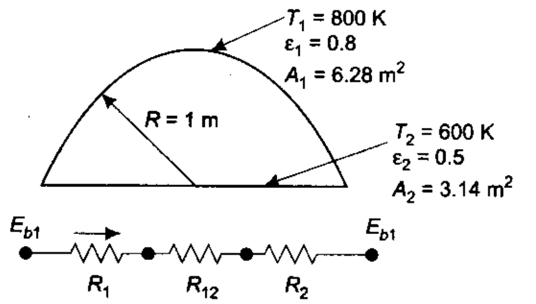

FIGURE Example 13.18 Radiation network for heat transfer between a hemispherical furnace and its floor

Solution. This is a two-zone enclosure problem. Fig. Example 13.18 shows the radiation network for this problem. We have:

$$Q_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

where, R_1 and R_2 are the two-surface resistances and R_{12} is the space resistance between the two radiosity potentials.

Data:

$T_1 := 800$ K $\quad T_2 := 600$ K $\quad \varepsilon_1 := 0.8$

$\varepsilon_2 := 0.5 \quad R := 1$ m

$A_1 := \frac{4 \cdot \pi \cdot R^2}{2}$ m² *(area of hemispherical surface 1)*

i.e. $A_1 = 6.283$ m² *(area of surface 1)*

$A_2 := \pi \cdot R^2$ m² *(area of surface 2)*

i.e. $A_2 = 3.142$ m² *(area of surface 2)*

$\sigma := 5.67 \times 10^{-8}$ W/(m²K) *(Stefan–Boltzmann constant)*

View factors:

$F_{21} := 1$ *(since all the heat radiated by surface 2 is intercepted by hemispherical surface 1.)*

Now, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ *(by reciprocity)*

Therefore, $F_{12} := \frac{A_2 \cdot F_{21}}{A_1}$

i.e. $F_{12} = 0.5$ *(view factor from surface 1 to surface 2)*

Resistances:

Now, $R_1 := \frac{1 - \varepsilon_1}{\varepsilon_1 \cdot A_1}$

i.e. $R_1 = 0.0398$ m⁻² *(surface resistance of inner surface)*

and, $R_2 := \frac{1 - \varepsilon_2}{\varepsilon_2 \cdot A_2}$

i.e. $R_2 = 0.318$ m⁻² *(surface resistance of outer surface)*

Also, $R_{12} := \frac{1}{A_1 \cdot F_{12}}$

i.e. $R_{12} = 0.318$ m⁻² *(space resistance between inner and outer surface)*

Therefore,

$R_{tot} := R_1 + R_{12} + R_2$

i.e. $R_{tot} = 0.676$ m⁻² *(total resistance between inner and outer surface)*

Also,

$E_{b1} := \sigma \cdot T_1^4 \quad E_{b1} = 2.322 \times 10^4$ W/m²

$E_{b2} := \sigma \cdot T_2^4 \quad E_{b2} = 7.348 \times 10^3$ W/m²

Then, net rate of heat transfer between surface 1 and 2 is given by:

$$Q_{12} := \frac{E_{b1} - E_{b2}}{R_{tot}}$$